A. F. COLLINGS AND E. MCLAUGHLIN

in benzene at 25°C for a crystal 5 cm long and 0.5 cm diam. having a resonant frequency of 39 kHz is approximately 416 nepers cm⁻¹. The amplitude of the wave is therefore reduced to e^{-1} of its initial value at a distance of 2.4×10^{-4} cm from the crystal surface. This distance is small compared to the crystal radius so that plane wave theory is applicable.

The degree of dampling in a viscous fluid can be readily measured in terms of an impedance loading on the crystal which produces a change in the crystal resistance at resonance, $\Delta R = R - R_0$ and in the resonant frequency, $\Delta f = f_0 - f$. For Newtonian fluids, these changes are related to the viscosity and density of the fluid by the equations:

$$R - R_0 = \Delta R = K_1 (\pi f \eta \rho)^{\frac{1}{2}},\tag{3}$$

$$f_0 - f = \Delta f = K_2 (\pi f \eta \rho)^{\frac{1}{2}}.$$
(4)

 K_1 and K_2 are electromechanical constants given by

$$K_{1} = \frac{2R(r^{-1} + l^{-1})}{\pi\rho_{0}(f_{2} - f_{1})} \left(1 - \frac{2\Delta f}{f_{0}}\right)$$
(5)

$$K_2 = (r^{-1} + l^{-1})/\pi \rho_Q, \tag{6}$$

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where r and l are the crystal radius and length respectively, and f_2 and f_1 are the frequencies above and below the resonant frequency where the crystal resistance is 2R. The half-conductance points, $(f_1 \text{ and } f_2)$, are clearly indicated in fig. 5. The factor $(1-2\Delta f/f_0)$ in eqn (5) is a correction for the increase in the effective moment of inertia of the crystal due to the reaction of the fluid, which is of the order of 0.1 % in the liquids investigated.

EXPERIMENTAL

FACTORS AFFECTING MEASUREMENTS

CRYSTALS AND LEADS

Extensive investigations of the torsional crystal technique for measuring viscosity have been made by Mason¹ who devised the method, by Rouse et al.² and Lamb and coworkers.³ In the present work the crystals used were supplied by Brush Clevite Co. and had a length of 5 cm and diameter of 0.5 cm. According to eqn (1), the fundamental mode of frequency corresponding to this length was about 39 kHz. The positioning of the electrodes on, and the attachment of the leads to, the crystal are shown in fig. 1. The quartz cylinders were cut to an accuracy in angle of 10' with the main axis parallel to the X-axis to obtain a torsional mode of vibration, which is favoured 3 by a length-to-diameter ratio of about 10:1. Four gold electrodes were formed by vacuum deposition in quadrants of 80° axially along the crystal with the unplated regions lying in the Y- and Z-axes. For the fundamental mode of vibration, the length of the crystal is equal to half a wave-length with a node at the centre of the crystal. The leads attached to the crystal at this point therefore had a minimum damping effect. No difference was found in using either 0.014" diam. copper or 0.008" diam. phosphor-bronze leads. S-bends were inserted in the leads between the crystal and a metal supporting frame to minimize strain on the lead attachments. Polished and unpolished crystals were used to examine the effect of the surface finish of the crystal.

CRYSTAL CONSTANTS

In theory, either the change in frequency or in resonant resistance can be used to determine the viscosity of the fluid when the crystal constants and fluid density are known. K_1 can be determined by calibration with liquids of known density and viscosity and use of eqn (3). Alternatively, the two half-conductance frequencies can be measured and used in

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(2)

p are the viscosity 1 wave, $(\pi f \rho / \eta)^{\frac{1}{2}}$, Sciences, Australian

n Rouge, Louisiana